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Stochastic testing of ADC—Step-Gauss method

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Abstract

The article describes theoretical background and practical results of Analog-to-Digital Converters (ADC) stochastic test method "Multi-Gauss" that has been designed, developed and verified at the Department of Measurement of the Czech Technical University, FEE in Prague. It is suitable for testing of high-resolution AD converters (e.g. $\Sigma-\Delta$ or dither-based) or on the contrary ultra high-speed AD converters. The method is based on the histogram test driven by stochastic signal with defined probability density function (p.d.f.). Further enhancement that allows an estimation of frequency dependency of effective number of bits (ENOB) is also presented.

Keywords: ENOB; ADC; Stochastic signal

1. Introduction

Low sampling frequency, high-resolution AD converters (e.g. $\Sigma-\Delta$ or dither-based) or on the contrary ultra high-speed AD converters (e.g. with opto-electronic core) testing by means of deterministic testing signal is a problem due to the lack of (pure) testing signals. In such cases, stochastic input signals seem to be better applied for testing [1,2], see also Fig. 1.

Histogram test can use various input signals and principally allows the use of noise as the input signal. To overcome difficulties related to generation of large-scale uniformly distributed stochastic signal, a method based on superposition of Gaussian noises with equidistantly spaced DC shifts (Fig. 2) has been proposed in Ref. [3] and theoretical analysis has been provided there. Practical applicability of this method has been verified by comparison of results obtained by this method and by standard histogram test using deterministic (ramp) testing signal for an internal Analog-to-Digital Converter (ADC) of digitizing oscilloscope [4], plug-in card for PC [5] and portable ADC transfer device [4–7]. One has to carefully design the test setup to obtain enough code words covered by each particular test signal.

2. Histogram stochastic test

Each particular testing signal is Gaussian noise with probability density function (p.d.f.) $f_G(\mu,\sigma)$

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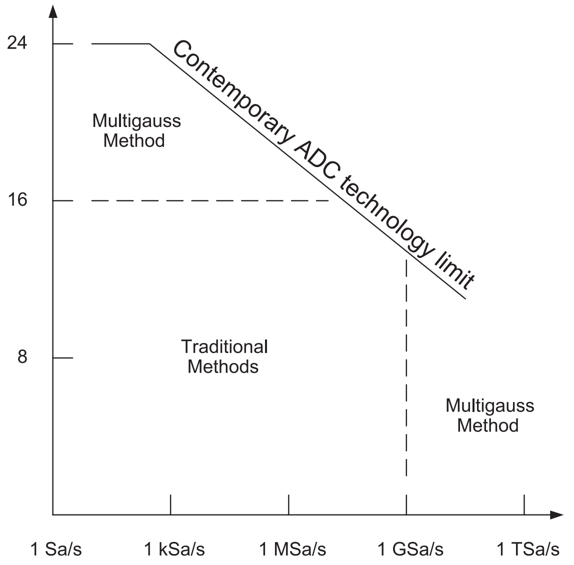

Fig. 1. Contemporary ADC from nominal number of bits vs. sampling frequency point of view and suitable testing methods.

where μ is the mean value and σ means standard deviation. It is easy to show that

$$\lim_{\Delta\to 0}\left[\left(\sum_{k=-\infty}^{\infty} f_{\mathrm{G}}(\mu+k\Delta,\sigma)\right)-\frac{1}{\Delta}\right]=0, k \text{ integer} \tag{1}$$

Independently on the value of μ and σ. In other words, the superposition of Gaussian distributed noises with the equidistantly spaced DC values by step Δ is for suitably small values of Δ, an excellent approximation of uniformly distributed signal. Measurement is provided for each Gaussian noise separately but histograms are cumulated, see Fig. 2.

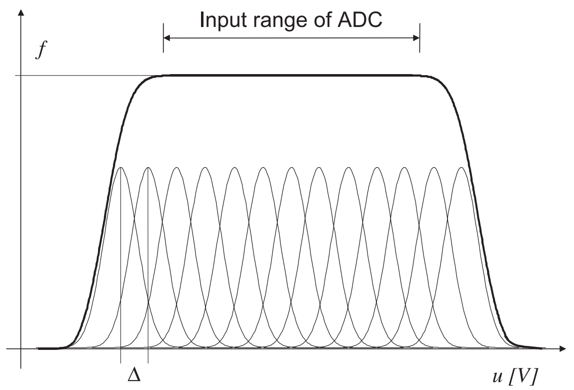

Fig. 2. Stochastic test signal generation.

Several factors must be taken into account to discuss the accuracy of the replacement of the resulting signal instead of uniformly distributed one. To discuss them, let

$$P=\frac{\Delta}{\sigma} \tag{2}$$

The parameter of the main interest is for sure the ripple R of p.d.f. $f(u)$ of resulting signal within the full-scale input range (FSR of ADC under test). This can be defined the following way:

$$R=\max_{u\in\mathrm{FSR}}\left\{\left|\frac{f(u)-\frac{1}{\Delta}}{\frac{1}{\Delta}}\right|\right\} \tag{3}$$

Then, the following factors should be analyzed:

- relation between ripple R and P;
- border errors—the difference between $f(u)$ and $1/\Delta$ values at the borders of FSR;
- influence of small changes of σ and μ that are considered to be constant during the whole measurement.

2.1. Influence of the step Δ on the ripple R

The dependency of ripple R on the ratio P is shown in Fig. 3 and listed in Table 1. No border error (as will

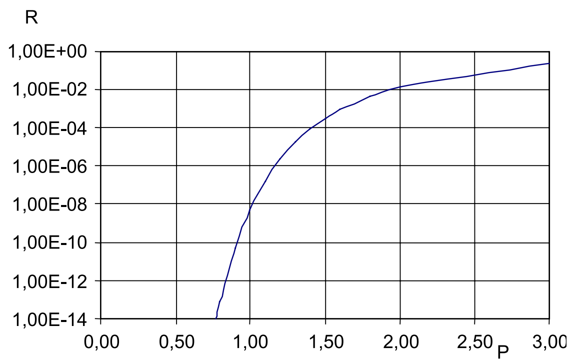

Fig. 3. The dependency of relative ripple R of resulting p.d.f. on the ratio P.

Table 1
The dependency of ripple of resulting p.d.f. on the ratio P

P	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.8	2	3
R	8.00e − 14	5.30e − 11	5.30e − 09	1.70e − 07	2.20e − 06	1.70e − 05	8.60e − 05	3.20e − 04	9.20e − 04	4.60e − 03	1.44e − 02	0.23e + 00

be discussed further) is expected. It is obvious that the ripple is approximately 1.4% for $P=2$ and less than 10^{-8} for $P<1$. For $P<1$, the value of ripple meets the requirements for testing up to 24-bit $\Sigma-\Delta$ ADCs.

2.2. Border errors

Let us define relative border error (BE) as

$$\mathrm{BE} = \max_{u\in\{\min(\mathrm{FSR}),\max(\mathrm{FSR})\}}\left\{\left|\frac{f(u)-\frac{1}{\Delta}}{\frac{1}{\Delta}}\right|\right\} \quad (4)$$

The dependency of BE on the number of Gaussian signals whose mean value override FSR has been investigated. We assume the symmetrical situation when the same number M of Gaussian signals is placed below minimum of FSR and above maximum of FSR. The results depend on the ratio P and are summarized in the Table 2:

2.3. Influences of variances of σ and μ

The values of σ and μ of each Gaussian signal are considered to remain constant during the particular measurement. However, small variances (such as drift of offset of summing amplifier causing changes of μ) must be assumed and its influence to the ripple R of resulting p.d.f. must be investigated. It has been found [3] that the following approximations are valid:

$$R_\sigma \approx 0.4 \times P \times \varepsilon_\sigma \quad (5)$$

and

$$R_\mu \approx 0.24 \times P^2 \times \varepsilon_\mu \quad (6)$$

where R_x means ripple caused by variances of x, $\varepsilon\sigma$ is relative error of σ (normalized by σ) and $\varepsilon\mu$ is relative error of μ (normalized by step Δ). For example, if one has a situation $P=1$ and if the error of equidistancy of DC shifts is 1% of the step Δ and the relative error of σ is 1%, too, the ripple R caused by equidistancy error is approx. 0.24% and the ripple caused by σ variations is 0.4%.

It is useful to note that in the case that particular Gaussian stochastic signals are generated by DAC, the used DAC should provide better accuracy than only the relevant portion of tested ADC. It means that it is not necessary to compare the linearity of the full scales of tested ADC and testing DAC.

Table 2
The dependency of relative ripple R of resulting p.d.f. on the number of overriding test signals M

M	BE ($P=2$)	BE ($P=1$)	BE ($P=0.5$)
0	0.09	0.3	0.4003
1	$<R$	0.057	0.2242
2	$<R$	0.0046	0.1032
3	$<R$	1.35e − 04	0.0385
4	$<R$	1.49e − 06	0.0115
5	$<R$	$<R$	0.0027
6	$<R$	$<R$	5.1e − 04
7	$<R$	$<R$	7.5e − 05
8	$<R$	$<R$	8.8e − 06
9	$<R$	$<R$	8.0e − 07

Symbol $<R$ means that for such combination of values R and M, the resulting BE would be smaller that the ripple R. Therefore, it makes no sense to choose such high value of M.

3. Required number of samples

The following formula should be used to calculate the necessary amount of samples to achieve a required accuracy [3]:

$$k = m\frac{a^2}{\varepsilon^2} \quad (7)$$

where k is number of required samples, m is number of code words of the tested ADC, $a=1.96$ for 5% confidence level of Differential Nonlinearity (DNL) evaluation and ε is the statistical error of DNL evaluation (5% in our case).

4. ENOB calculation

Effective number of bits (ENOB) calculation follows the usual way that is described in Ref. [2] or in

Ref. [3]. In the concrete, DNL values are estimated from the measured histogram and Integral Nonlinearity (INL) values are then calculated using DNL:

$$\mathrm{DNL}_i = \frac{O_i - O}{O} \tag{8}$$

$$\mathrm{INL}_j = -\sum_{i=1}^{j} \mathrm{DNL}_i \tag{9}$$

where O_i is the value of the i-th code of cumulative histogram and O is its ideal value that can be achieved by the following way (the common way of indexing of code words $0,1,\ldots,m-1$ is expected):

$$O = \frac{\sum_{i=1}^{m-2} O_i}{m-2} \tag{10}$$

Assuming statistical independence of INL values and quantisation error, the standard deviation of ADC output can be calculated as

$$\sigma_c = \sqrt{\frac{1}{12} + \frac{1}{m-2}\sum_{i=1}^{m-2} \mathrm{INL}_i^2} \tag{11}$$

Then, ENOB can be calculated as

$$\mathrm{ENOB} = \log_2 \frac{m}{\sigma_c\sqrt{12}} \tag{12}$$

4.1. ENOB frequency dependency estimation

It is possible to obtain set of values of integral ENOB for particular (narrow) frequency bands by including tunable band-pass filter to the input signal

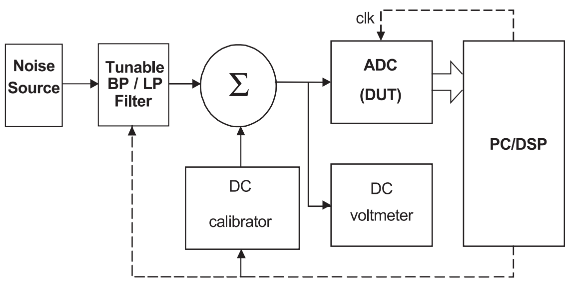

Fig. 4. Arrangement of testing system for ADC stochastic testing.

Table 3
Test results for internal flash ADC of digital oscilloscope HP 54645A (200 MSa/s)

Frequency	Deterministic (harmonic) signal		Stochastic signal	
	$\mathrm{ENOB}_{\mathrm{DFT}}$	$\mathrm{ENOB}_{\text{sine-fit}}$	$\mathrm{ENOB}_{\text{hist-sin}}$	$\mathrm{ENOB}_{\text{Step-Gauss}}$
500 kHz	6.90	7.02	7.05	7.20
1 MHz	6.92	7.01	7.10	7.00
2 MHz	6.90	7.05	7.08	7.06

path and by procedure repetition for different settings of middle filter frequency. Description of this procedure and relevant relations, as well as practical results obtained on digital oscilloscope and PC plug-in ADC board has been provided in Ref. [5]. In this case, the variance of (wide-band) noise source has to be high enough, or, in different words, the step of DC calibrator (see Fig. 4) has to be small enough.

To overcome this difficulty of the method, further modification has been developed [7]. It substitutes band-pass filter that reduces signal variance significantly with low-pass tunable filter. In this case, it is necessary to recalculate output values in the following way. Expecting

$$\sum_{i=0}^{j} \Delta f_i = f_j \tag{13}$$

$$\mathrm{ENOB}_{\mathrm{LP}}(f_j) = \frac{1}{f_j}\sum_{i=0}^{j} \Delta f_i \mathrm{ENOB}(f_i) \tag{14}$$

Table 4
Test results for ADC plug-in card AD14DSP (250 kSa/s)

Low-pass filter		Equivalent narrow-band ENOB: Recalculation according to Eq. (18)	
f_{LP} (kHz)	$\mathrm{ENOB}_{\mathrm{LP}}$	f_{BP} (kHz)	$\mathrm{ENOB}_{\mathrm{BP}}$
2	12.08	2	12.08
5	12.03	3.2	12.00
10	11.96	7.1	11.89
50	11.67	22.4	11.60
100	11.56	70.7	11.45
200	11.37	141	11.18
300	10.94	245	10.08

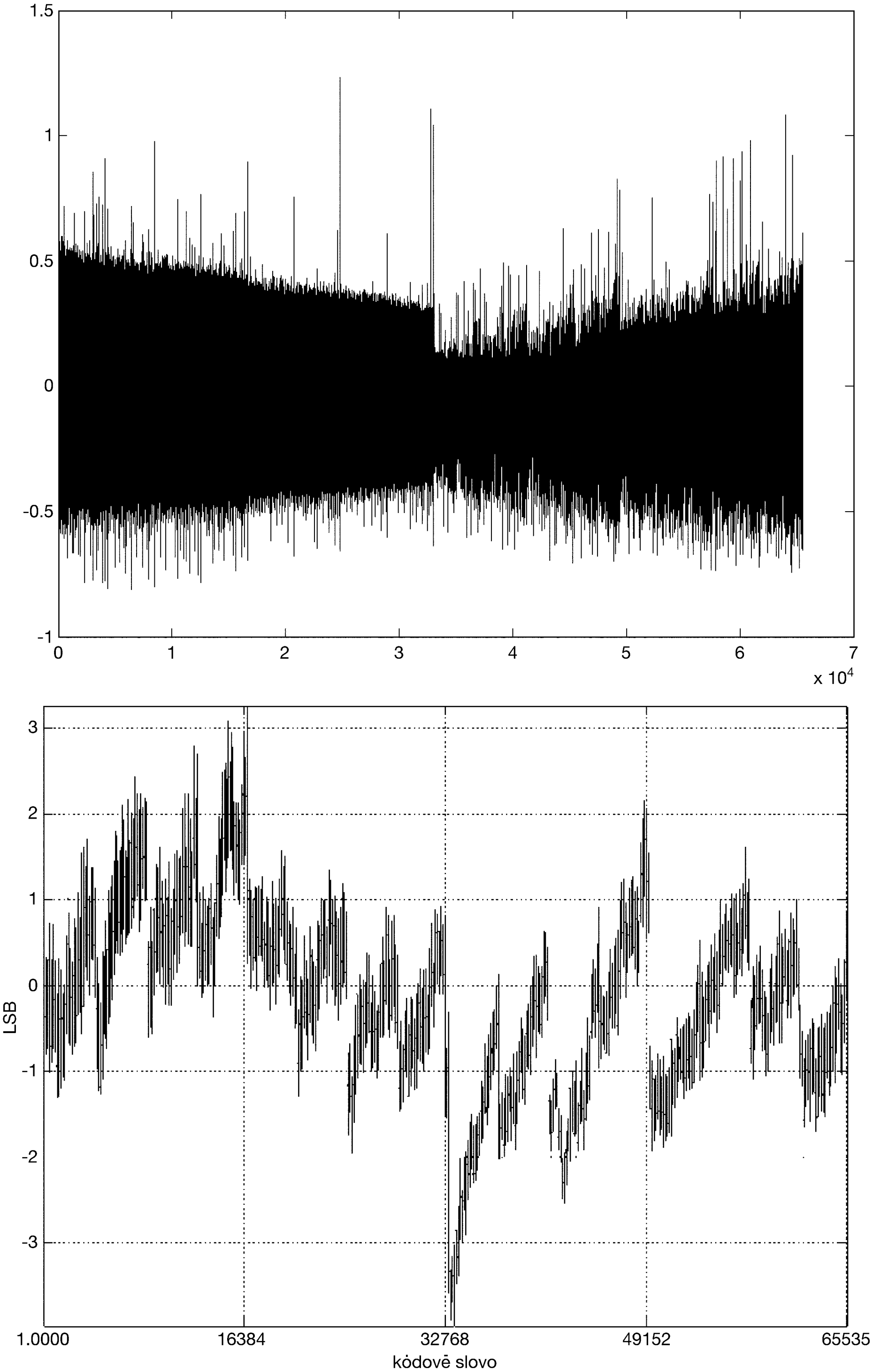

Fig. 5. Differential (DNL, upper picture) and Integral Nonlinearity (INL, lower picture) of the tested AD976A [LSB] using Step-Gauss stochastic test method.

where ENOB(f_i) represents measurement results for pass-band filter of mean frequency f_i and bandwidth Δf_i application. Then follows

$$\mathrm{ENOB_{LP}}(f_j) = \frac{1}{f_j}\left[\sum_{i=0}^{j-1} \Delta f_i \mathrm{ENOB}(f_i) + \Delta f_j \mathrm{ENOB}(f_j)\right] \tag{15}$$

$$\mathrm{ENOB}(f_j) = \frac{1}{\Delta f_j}\left[f_j \mathrm{ENOB_{LP}}(f_j) - \sum_{i=0}^{j-1} \Delta f_i \mathrm{ENOB}(f_i)\right] \tag{16}$$

$$\mathrm{ENOB}(f_j) = \frac{f_j}{\Delta f_j}\mathrm{ENOB_{LP}}(f_j) - \frac{f_{j-1}}{\Delta f_j}\mathrm{ENOB_{LP}}(f_{j-1}) \tag{17}$$

For wide bandwidths (f_{i-1}, f_i), it is more accurate to write $\sqrt{f_i f_{i-1}}$ as index of the result:

$$\mathrm{ENOB}\left(\sqrt{f_j f_{j-1}}\right) = \frac{1}{\Delta f_j}\left[f_j \mathrm{ENOB_{LP}}(f_j) - f_{j-1}\mathrm{ENOB_{LP}}(f_{j-1})\right] \tag{18}$$

where ENOB() means effective number of bits estimation in the frequency band (f_{j-1},f_j), ENOB$_{LP}$(f_i) is the integral effective number of bits obtained by application of low-pass filter with cut-off frequency f_i. Uniformly distributed power spectral density of input stochastic test signal is expected.

5. Measurement results

Two tested objects have been selected to confirm practical applicability. The results of testing of digital oscilloscope HP54645A are shown in Table 3. Since the testing system operates up to several MHz and the sampling rate of the internal flash 8-bit ADC is 200 MSa/s sample rate, no differences have been found. Small variations of ENOB values shown in Table 3 fall within uncertainty level that is given by finite number of samples, as indicated by formula (7). Practical applicability of this method has been verified by comparison of results obtained by this method and by standard histogram test using deterministic testing signals.

The results of oscilloscope testing are useful since results of deterministic testing by histogram method driven by harmonic signal are available (see Table 3). The differences in evaluated ENOB are less than 2% in all cases.

ADC plug-in board has been tested as the second object. The results are given in Table 4. As the summing amplifier, the AD825-based circuit has been used. The bandwidth (−3 dB) of summing circuit is greater than 10 MHz. The following measuring equipment was used: Stanford Research signal generator DS360 as the noise generator and Fluke 5100B as a DC voltage source. According to Eq. (7), 10^8 samples are necessary for Step-Gauss test of 16-bits ADC to achieve 5% confidence level of DNL evaluation and the same statistical error of DNL evaluation. Due to nature of Step-Gauss test, this amount of samples has been split into 44 blocks with different DC offsets of test Gauss signal.

The result differences between different tests fit easily into uncertainty interval. However, they confirmed well-known rule that sine-fit test provides lower ENOB due to the fact that also input and internal noise are considered, while for long-record-based FFT test and for all histogram-based methods, those types of noise are filtered out.

The applicability of Step-Gauss stochastic test method has been confirmed also on 16-bit, 200 kSa/s successively approximating ADC AD976 housed in portable reference device. It enables to perform comparative measurements even for stochastic tests, however, it is necessary to carefully design the test conditions to balance the requirements for high number of samples necessary for stochastic tests and limited data transfer capability of the device, see Fig. 5 and Table 5.

Table 5
Test results for AD976A [LSB] using Step-Gauss stochastic test method

Frequency (Hz)	ENOB$_{FFT}$ (bit)	ENOB$_{sine\text{-}fit}$ (bit)	ENOB$_{Step\text{-}Gauss}$ (bit)
2333	13.79	13.80	13.88
5333	13.96	13.97	13.88
10 333	13.98	14.00	13.92
20 333	13.92	13.92	14.01
50 333	13.63	13.62	13.88
100 333	13.21	13.20	13.65

6. Conclusions

A special type of stochastic testing signal for ADC tests has been introduced. It allows substitution of difficult-to-generate but easy-to-process uniformly distributed stochastic signal with a cumulation of Gaussian signals that are easy to generate. Two other modifications of this method are presented. They allow estimation of the dependency of effective number of bits on the frequency content of input signal. Experimental results that confirm the method applicability are available.

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