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Digital filtering for noise reduction in nuclear detectors

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ABSTRACT

Specific algorithms for the elaboration and the digital filtering of signals generated in nuclear-particle detectors have been studied and optimized. These algorithms will contribute to build up a data acquisition system to be drawn on the next generation of hadron physics experiments. Working in a non-stationary environment, adaptive filters can optimize in a dynamic way the retrieval of information in the tracking of charged particles. In this paper, the performances of different simulated filters are presented, and their simulations discussed.

1. Introduction

The standard architecture of data acquisition systems (DAQ) in nuclear detectors is based on a two-layer hierarchical approach. A subset of especially instrumented detectors is used to evaluate a first-level trigger condition. For the accepted events, the full information of all detectors is then transported to the next higher trigger level or to storage. The time available for the first-level decision is usually limited by the buffering capabilities of the front-end electronics [1–3].

The next generation of experiments in the hadron facilities, like the FAIR one at Darmstadt [4], will study rare events at a drastically improved sensitivity. Interesting signals will only become available by a combination of high interaction rates (normally higher than 10 MHz), fast detectors and broad bandwidth data acquisition systems to select in a fitting way only the events of interest. These constraints make it necessary to go beyond the old two-layer hierarchical approach towards self-trigger systems. They autonomously detect signals and preprocess them to extract and transmit only the physically relevant information, marked by a precise time stamp and buffered for further processing. This means that they are able to discriminate how relevant the event is and, if required to select it, to select means to filter in the right way and dynamically the signals (see for example Ref. [5]).

This research unit aims to develop a data acquisition system through the study of specific algorithms for the reliable detection of "informative" pulses (i.e., the pulses generated by the interaction of a charged particle) partially buried in noise, as well as their implementation on electronic boards which use programmable devices.

As a starting point, the performance of a set of standard digital filters for signal denoising (Low-Pass (LP): Bessel, Butterworth, Chebyshev) have already been studied and characterized. However, using this kind of filters the Signal to Noise Ratio (SNR) cannot be sufficiently enhanced. Dealing with a non-stationary environment, adaptive filters look as a better choice to solve our problem. We have focussed on the application of Least Mean Square (LMS) adaptive filter [6,7,9]. The filtering systems have been studied, modelled, and simulated with specific programming languages (MATLAB, SIMULINK [10]).

The paper is organized as follows. In Section 2, the system model is introduced. Simulation scheme is presented in Section 3. In Section 4, standard and adaptive algorithms are described. The comparisons are dealt in Section 5. The paper is concluded with a discussion on the obtained results and an outlook to further developments in Section 6.

2. System model

When a charged particle is detected, the detector produces a current pulse that is processed through a transmission chain. This signal is affected by several causes of noise (thermal, shot, flicker, etc.) which impair the correct pulse detection.

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We aim to develop a filtering system able to reduce this noise as much as possible. The elaboration will be performed in the digital domain and for this reason the signal must be processed by an Analog to Digital Converter (ADC). However, because of the reduced input bandwidth (20 MHz) and the sampling rate of the ADCs currently available, the bandwidth of the input signal (i.e., informative pulses plus noise) has to be properly reduced. Thus, a low-pass analog transmission chain had to be introduced before the ADC [11]. A possible model is presented in Fig. 1, together with a pictorial representation of the impulse response.

The analog section is composed of the following:

- *Detector*: it detects charged particles and produces informative pulses with amplitude proportional to the charge (Q).
- *Preamplifier/integrator*: it integrates the input signal and at the same time reduces the bandwidth. When the current pulse is present, the integrator produces a signal proportional to the charge. The gain k_p is used to normalize the peak to a fixed value; τ_p is the time constant of the preamplifier/integrator.
- *PoleZero compensator*: it introduces a faster pole erasing the preamplifier one, in order to enlarge the total pass-band and thus to avoid as much as possible distortion of the informative pulses, which can cause a pile-up effect of the filtered pulses. The gain k_{pz} is used to normalize the peak to a fixed value, while the time constant $\tau_{pz} \ll \tau_p$.
- *Analog shaper/antialias filter*: it represents the final LP anti-aliasing filter, opportunely matched to the ADC input bandwidth and sampling rate. In practice, it enlarges the top of the signal, allowing the ADC to obtain more than one significant sample for each informative pulse. The gain k_{AS} is used to normalize the peak to a fixed value, $\tau_{AS} = 1/f_B$ is the time constant, where f_B is the desired signal band, and n is the denominator exponent. The factor n determines the slope of the transfer function in the transition bandwidth and the top flatness of the signal. However, the higher the parameter n is, the more difficult the hardware assemblies are. Hence, n was chosen as a compromise between the hardware complexity and the minimization of the sampling error.
  After these three blocks the filtered signal is ready to be elaborated by the digital sub-chain (Fig. 2)
- *ADC*: it converts the analog signal into a digital one.
- *Noise filter*: it is digital and is designed to possibly reduce the noise that affects the desired signal. It can be standard or adaptive, with Finite (FIR) or Infinite (IIR) Impulse Response.

3. Simulation scheme

In order to evaluate the improvement obtained using a digital filter to partially suppress the noise, in our simulation we modelled the information pulses with a series of successive finite support waveforms, with very narrow time duration (0.5 ns) and

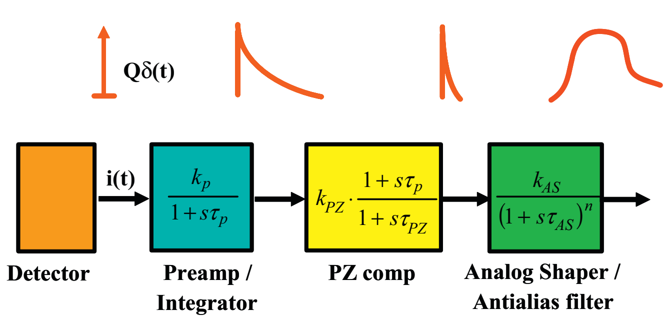

Fig. 1. Analog transmission sub-chain.

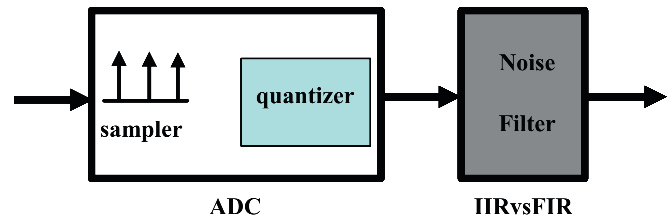

Fig. 2. Digital transmission sub-chain.

random times of arrival. No specific assumptions have been made at this stage about the mathematical model of the times of arrival, since experimental results are not still available. However, superimposed input pulses have been explicitly avoided. The pulses series are processed by the transmission chain model and the selection of the noise model has been essentially driven by the sake of simplicity. Thus, an additive White Gaussian Noise (WGN) model has been chosen and the addition stage has been placed right before the ADC device, as shown in Fig. 3. This can represent a situation where the noise has a significantly wider bandwidth than the signal of interest (pulses).

We evaluated by simulation the behaviour of several digital filters (using MATLAB, SIMULINK [10]). In particular, we focused on

- standard LP III order Butterworth filter;
- adaptive LMS filter.

Aiming at the best noise reduction, we compared the output of every digital filter to the digitalized output of the analog shaper. The results are presented in the following section.

4. Digital conversion and filtering algorithms

Let us consider the detection of two charged particles. The detector will produce two short current pulses with the amplitude proportional to the charge carried by every single particle.

In our simulation the two amplitudes are different, the first is equal to 1 (normalized current unit) and the second to 0.5; the pulse duration is 0.5 ns, while the interarrival time is 0.7 μs. The bandwidth used to digitally represent the analog section is 5 GHz, while the digital bandwidth after the ADC is reduced to 50 MHz.

The impulse response of the analog blocks is summed up in Fig. 4.

Adding to the shaper output a WGN signal one-sixth less powerful at the same sampling rate, we obtain a simulation of a noisy analog measurement (Fig. 5).

Putting our attention on the analog shaper output, Fig. 5a represents the desired signal we aim to extract from a noisy measurement (Fig. 5b), after sampling and quantization. Note that the sampling operation is a simple down-sampling in our simulation. Quantization is performed by the model of ADC introduced in Section 3.

Performing the Discrete Fourier Transform (DFT) of the digitalized analog shaper output, we have its representation in the frequency domain. Its square modulus provides the Power Spectral Density (PSD) of the considered signal; it is a useful mathematical tool that shows the most important frequency components in the signal spectrum.

In Fig. 6 we can see the PSD of the analog shaper output, where the signal power has been normalized to the unit. The most significant frequencies are bounded in the lower part of the spectrum; as a consequence, the starting point of our analysis is a

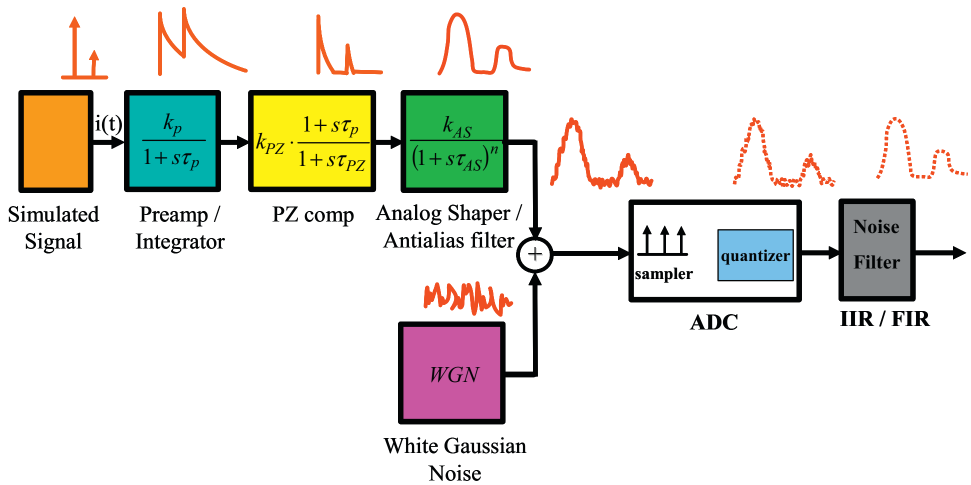

Fig. 3. Simulated transmission chain.

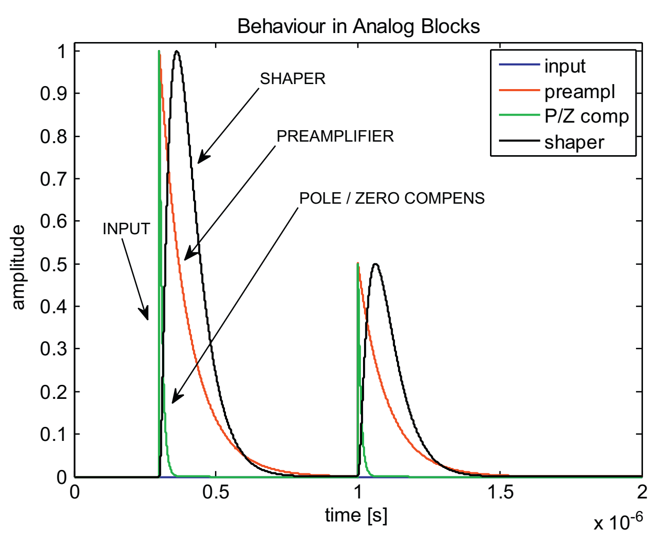

Fig. 4. Behaviour in analog blocks.

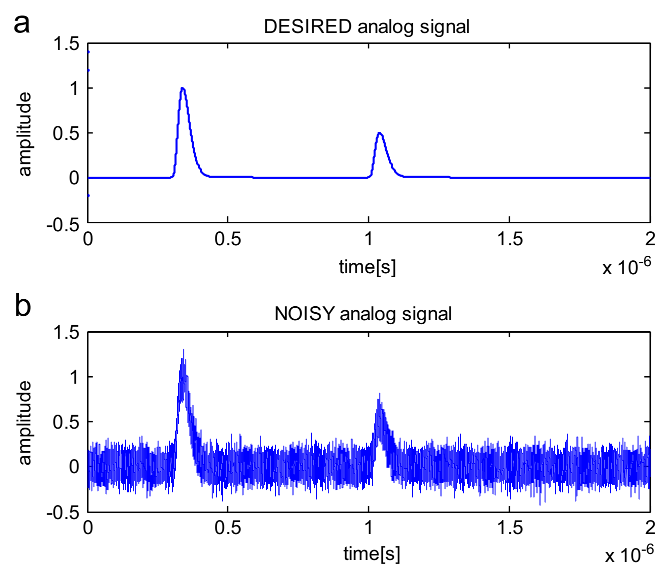

Fig. 5. Continuous shaper output: desired vs. noisy.

LP digital filter. Furthermore, Shannon Theorem is satisfied because the band involved is 20 MHz and we sample this signal with a sampling frequency of 100 MHz ($>2\times 20$ MHz).

4.1. Butterworth LP digital filter

For the sake of comparison, the SNR and the "peak reproducibility" are reported in Tables 1 and 2 where different standard filter families and orders are considered. The SNR is calculated as the ratio between the desired signal power and the noise power, expressed in dB. The "peak reproducibility", or percentage error, is defined as the relative difference between the desired signal peak and the filtered one, expressed in percentage. In case of several peaks, we decided to focus on the case with the highest distortion. The percentages shown in Table 2 all refer to the same worse condition.

From these characterizations the best performances both in noise reduction and in peak reproducibility are obtained with the Butterworth family. The highest noise reduction is obtained with the III order transfer function, while the highest peak reproducibility (lowest percentage error) is obtained with the II order.

We chose the III order because our aim was the noise reduction and we set the cut-off frequency to 20 MHz, i.e., to the bandwidth of the input signal. The normalized transfer function in the continuous complex frequency domain is

$$H(s)=\frac{1}{s^3+2s^2+2s+1}.$$

To convert this analog transfer function in the digital domain, we used a digital transfer function $H(z)$ obtained from bilinear transform of $H(s)$ [12].

A typical implementation of the digital IIR filter corresponding to $H(z)$ is the direct form II, where $x(n)$ is the input noised signal and $y(n)$ the output filtered signal (Fig. 7).

4.2. MMSE noise canceller

The standard III order LP Butterworth analog filter has fixed parameters (transfer function coefficients) that are known and calculated with Butterworth III order polynomials [8], denormalized at a particular cut-off frequency, bilinear transformed and then implemented in the digital filter. However, they

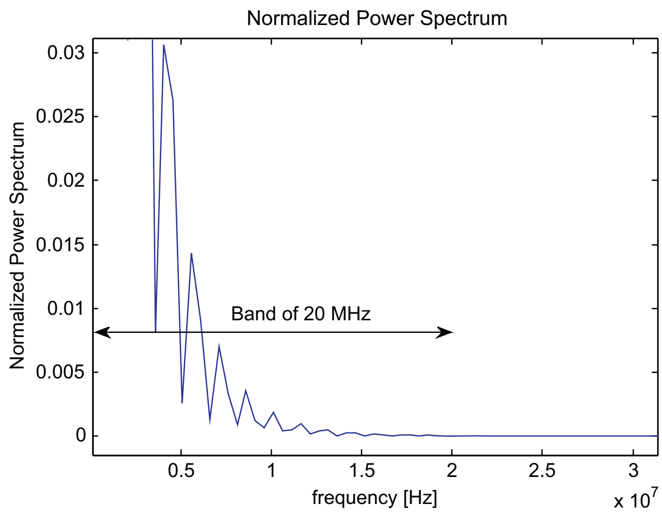

Fig. 6. PSD of analog shaper output.

Table 1
SNR improvement [dB], our choice in bold typeface

Order	II	III	IV	V
Butt.	5.50	**5.76**	5.88	5.95
Bess.	4.22	3.52	3.05	2.71
Cheb.	2.88	4.29	4.95	5.37

Table 2
Peak reproducibility [% error], our choice in bold typeface

Order	II	III	IV	V
Butt.	7.72	**8.34**	10.45	11.40
Bess.	5.89	7.19	7.93	8.38
Cheb.	7.45	9.60	12.66	12.58

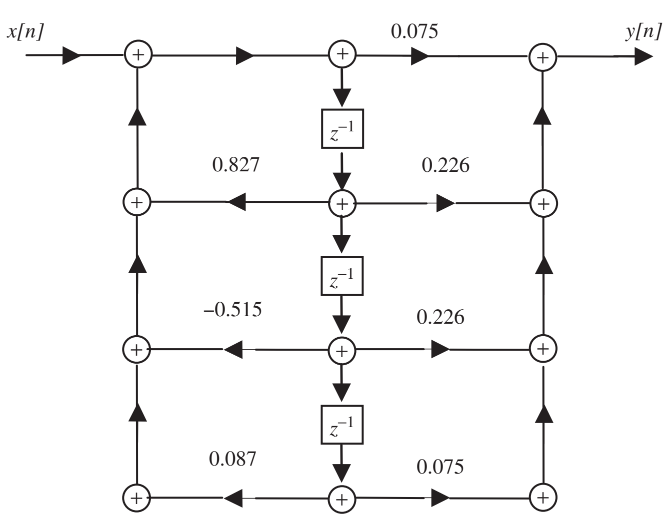

Fig. 7. Digital Butterworth III implemented with direct form II.

do not depend on the characteristics of the specific considered signal. In this section, we introduce an algorithm whose parameters are dynamically calculated and adapted to the input signal in real time, the LMS algorithm (see Ref. [7] for a complete development).

We are interested in the Minimum Mean Square Error (MMSE) noise canceller implementation of this filter and in its Finite Impulse Response (FIR) form. If a process $d(n)$ is to be estimated from an observed process $x(n)$ corrupted by the noise $v(n)$

$$x(n) = d(n) + v(n)$$

and if we do not have any kind of information about $d(n)$ or $v(n)$, it is not possible to separate the signal from the noise. However, given a reference signal, this problem can be solved [7]. In the nuclear detector applications, here considered, we cannot have a reference signal and so we can adopt a different approach. We simply delay the process $x(n)$ of n_0 samples (Fig. 8), where $x(n)$ is the measurement, $d(n)$ is the desired component, $\hat{d}(n)$ is an estimate of d(n), $v(n)$ is the noise component uncorrelated from $d(n)$, $\hat{v}(n)$ is an estimate of $v(n)$.

In our model we can assume that $d(n)$ is a narrowband process (Fig. 6) and that $v(n)$ is a broadband process with

$$E\{v(n) \cdot v(n-k)\} = 0, \quad |k| \geqslant k_0$$

where $E\{\cdot\}$ is the statistical expectation and, if $v(n)$ is white, then $k_0 = 1$. So shifting the reference of at least k_0 samples, the noise component of the signal $x(n{-}n_0)$ is uncorrelated with the noise of the measured signal $x(n)$.

Therefore, if $k_0 \leqslant n_0 \leqslant k_1$, the delayed process $x(n{-}n_0)$ will be uncorrelated with the noise $v(n)$, but correlated with $d(n)$ (from the condition $n_0 \leqslant k_1$). Thus, the samples of $x(n{-}n_0)$ may be used as a reference signal to estimate $d(n)$ as illustrated in Fig. 8.

The problem we want to solve is how to obtain an estimate of the current sample $d(n)$ of the desired signal from a set of $M = k_1{-}k_0{+}1$ previous samples of the measured signal:

$$\bar{x}_M(n-n_0) = [x(n-n_0), x(n-n_0-1), \ldots, x(n-n_0-M+1)]^{\mathrm{T}}.$$

To do this, the observation vector $\bar{x}_M(n{-}n_0)$ must be filtered by a proper "linear predictor", designed as a FIR filter with coefficients

$$\bar{w}_M = [w_0, w_1, \ldots, w_{M-1}]^{\mathrm{T}}$$

so that the filtered signal, $\hat{d}(n)$, is written as

$$\hat{d}(n) = \bar{w}_M^{\mathrm{H}} \cdot \bar{x}_M(n-n_0) = \sum_{k=0}^{M-1} w_k^* \cdot x(n-n_0-k)$$

where $\bar{w}_M{}^{\mathrm{H}}$ is the Hermitian transpose of $\bar{w}_M$.

The optimum design of the filter coefficients can be made through the minimization of the Mean Square Estimation Error (MSE) [12], defined as

$$E\{|e(n)|^2\}$$

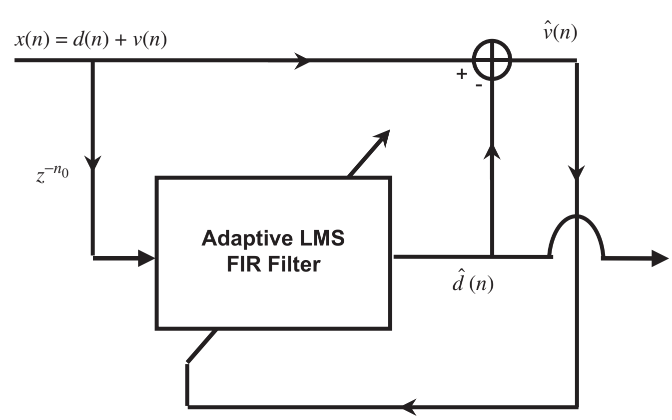

Fig. 8. MMSE noise canceller using a LMS filter.

where $e(n) = d(n) - \hat{d}(n)$ is the estimator error. It is a known result of the MMSE filter design theory [7,9] that the optimum set of filter coefficients for the linear prediction problem, stated as before, is given by

$$\bar{R}_{xx} \cdot \bar{w}_M = \bar{r}_{xd}$$

where $\bar{R}_{xx} = E\{\bar{x}_M(n-n_0) \cdot \bar{x}_M{}^{H}(n-n_0)\}$ is the $M \times M$ autocorrelation matrix of the input process, $\bar{x}_M{}^{H}$ is the Hermitian transpose of $\bar{x}_M$, and $\bar{r}_{xd} = E\{\bar{x}_M(n-n_0) \cdot x(n)\}$ is the cross-correlation vector between the past observation $\bar{x}_M(n-n_0)$ and the current one $x(n)$, which contains the desired component d(n).

However, in the case of the considered experiments, the non-stationarity of the observed process suggests to choose an iterative, adaptive implementation of the above formulation, known as LMS adaptive filter [7,9].

Using a one-point sample mean (for a more complete discussion of the LMS algorithm the reader is referred to Ref. [9]), the update equation assumes a simple form known as the LMS Algorithm:

$$\bar{w}_M(n+1) = \bar{w}_M(n) + \mu \cdot e(n) \cdot \bar{x}^{*}_M(n-n_0) \qquad (1)$$

where $\bar{w}_M(n+1)$ is a new vector of filter coefficients at time n+1, $\bar{w}_M(n)$ is the filter coefficients vector at time n, $e(n)$ is the error at time n, $\bar{x}^{*}_M(n-n_0)$ is the complex conjugate of the measurement at time $n-n_0$, n_0 is the introduced delay, and μ is the step size. It is a positive number that affects the rate at which the weight vector $\bar{w}_M(n)$ moves down towards a stable solution.

Since this work is preliminary for the implementation of these filtering algorithms on FPGAs, we need to take into account their computational complexity. Let us consider the LMS complexity in terms of additions and multiplications: Eq. (1) requires one addition to compute the error $e(n)$ and one multiplication to form the product $\mu e(n)$, M multiplications, and M additions to update the filter coefficients. Finally, M multiplications and $M-1$ additions are necessary to calculate the output, $y(n) = \hat{d}(n)$, of the adaptive filter. Thus, a total of 2M+1 multiplications and 2M additions per output point are required.

The choice of the step size μ corresponds to a tradeoff among:

- the *SNR*, in order to have a rough estimation of the real noise reduction
- the time dependence of the *squared error function* ($e^2(n)$). The $e(n)$ function is a positive or negative quantity involved in Eq. (1) responsible for real time correction of the filter coefficients. If the algorithm converges to a stable set of coefficients the correction, as a function of time, and its squared estimate, should have a decreasing behaviour.
- the *coefficient settlement* during the measurement or the simulation in order to understand if the algorithm has reached a stable solution.

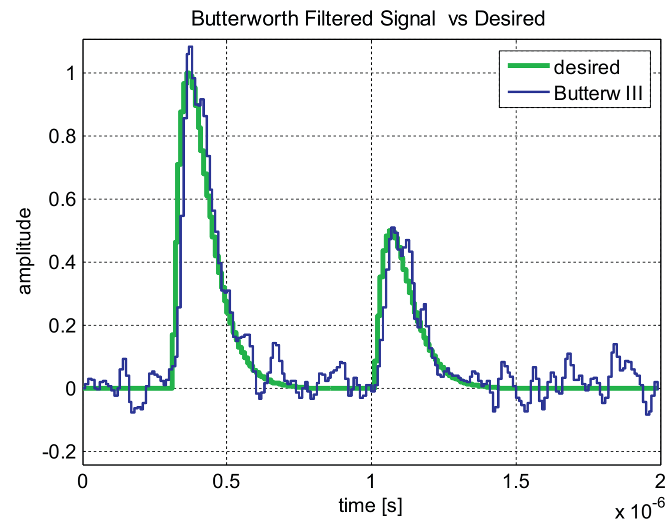

Fig. 10. Butterworth III order filtered signal vs. desired signal.

Table 3
LMS performances vs. μ, best result in bold typeface

LMS	step size μ	% error	SNR improvement (dB)
	0.05	36.88	5.51
	0.10	15.54	6.35
	0.15	5.70	6.58
	0.18	0.**06**	6.**57**
	0.20	2.63	6.52
	0.25	5.93	6.33
	0.30	6.67	6.09
Butterworth		8.34	5.76

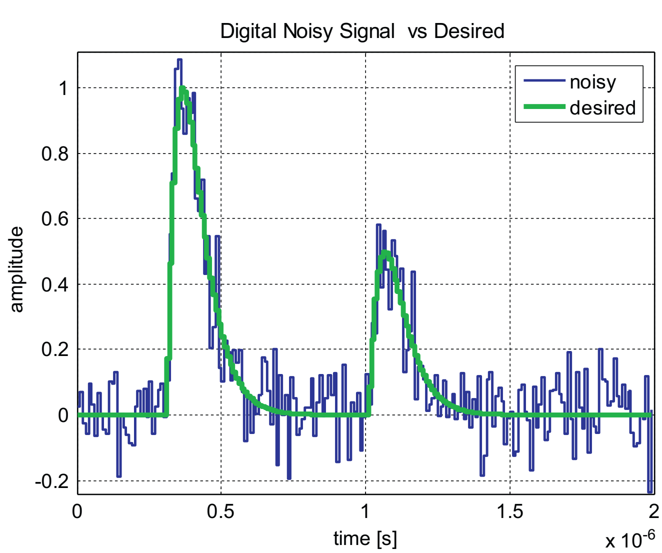

Fig. 9. Noisy vs. desired digital signal.

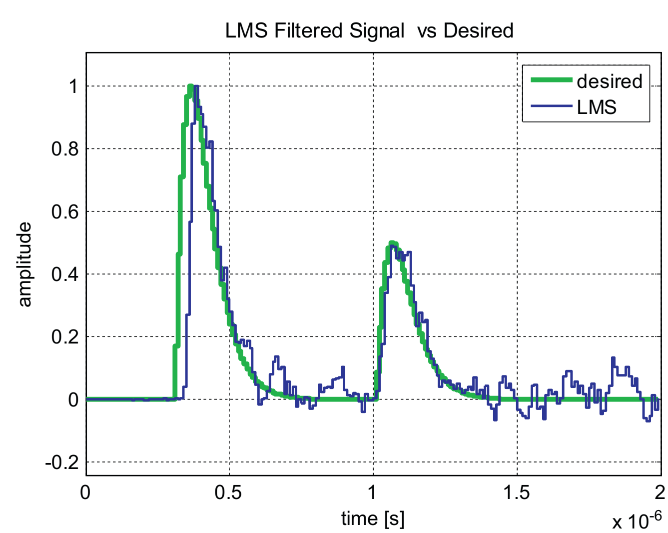

Fig. 11. LMS filtered signal ($\mu = 0.18$) vs. desired signal.

5. Behaviour comparison and discussion

In Fig. 9 the desired digital signal is plotted superimposed on the noisy digital signal.

The filtered signal obtained with an IIR Standard Butterworth LP III order digital filter is presented in Fig. 10.

The input SNR is 8.41 dB. Using the Butterworth filter this quantity arises to 14.17 dB with an improvement of 5.76 dB. However, this filter introduces a peak amplitude distortion that is worse for the first of the two processed pulses. In Fig. 10, the amplitude of the first peak for the Butterworth filtered signal is greater than the desired signal, being the distortion of 8.34%.

The order n of the FIR LMS was fixed to $n = 4$ to introduce a medium level of complexity with a reasonable elaboration time. Using a LMS filter the performances are a function of the step size μ, as shown in Table 3. The percentage error is calculated as for the Butterworth filter for the first peak, that is for the highest distortion condition.

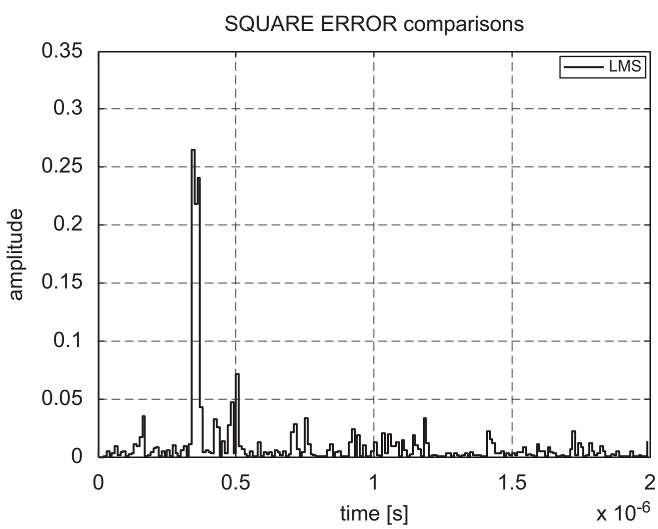

Fig. 12. Squared error vs. time: decreasing behaviour.

In Fig. 11, the LMS filtered signal with $\mu = 0.18$ is presented; the output SNR is 14.98 dB, so the enhancement is of 6.57 dB, and the peak reproducibility is also enhanced (lower percentage error). Indeed, in Fig. 11 the amplitude of the first peak for the LMS filter nearly matches the desired signal, the distortion being only of 0.06%.

The choice of the step size corresponds to a tradeoff among the evaluated SNR enhancement, squared-error function decreasing behaviour (Fig. 12), and coefficients settlement (Fig. 13) introduced in the previous section.

6. Conclusion and outlook

Two classes of digital filters, the Butterworth one and the adaptive LMS filter, have been compared by simulation. This study was originated by the need to extract the signal features for further on-line processing. The field of application is a new data acquisition system for nuclear physics experiments where the selection of the accepted events is no more performed by hardware triggers but is based on a sophisticated software system working on pre-processed data.

The requirements on peak reproducibility and SNR are matched better by the LMS filter, thanks to the capability to adapt the parameters to the typically non-stationary environment of the nuclear physics detectors. The simulation showed that a fast settlement of the coefficients can be reached with an algorithm of medium-level complexity, with an elaboration time scaling linearly with the filter order.

The LMS filter system considered here will be implemented on a FPGA for test on data streams coming from multi-channel systems operated at high rates (of the order of 10 MHz) with a programmable pulse generator.

Data sets of real signals will be used as well to study the time and energy resolution achievable in comparison with the algorithms currently used for the signal features extraction.

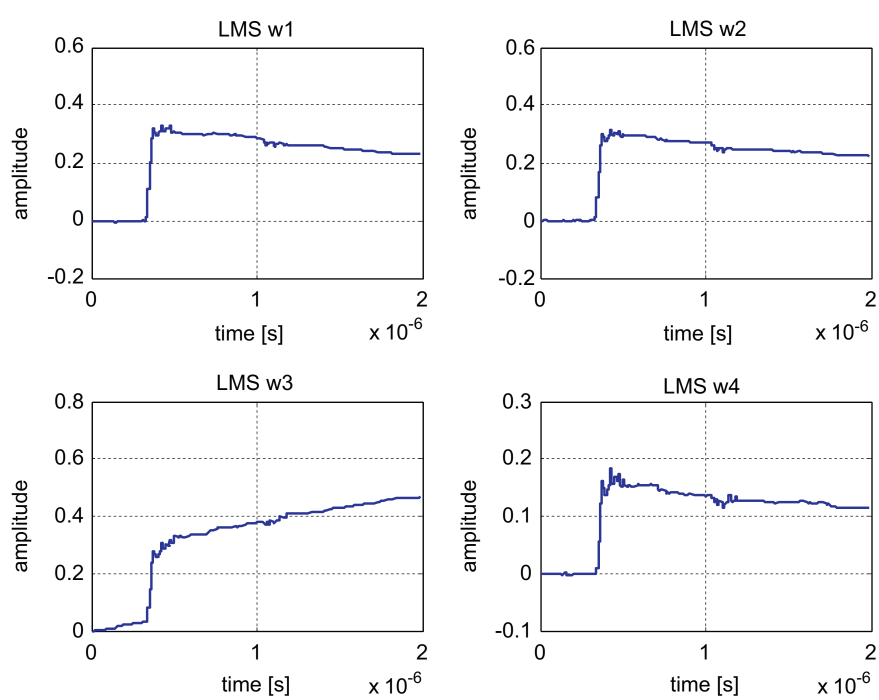

Fig. 13. LMS four coefficients ($\mu = 0.18$) behaviour: a stable solution is reached.

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